12/8/16 "After the game, the king and the pawn go into the same box." -Italian Proverb

HW: "Remainder and Factor Theorem" worksheet #Exercise Set A #1a-d
Test 2 on Tuesday 12/20

AIM: What are the Remainder and Factor Theorems?

Warm Up:
On Handout

No x^2

$$Let\ P(x) = 3x^5 + 5x^4 - 4x^3 + 7x + 3$$

(a) Find the quotient and remainder when P(x) is divided by x+2

$3x^4 - 1x^3 - 2x^2 + 4x - 1 + \frac{5}{x+2}$

$x+2 \overline{)\ 3x^5 + 5x^4 - 4x^3 + 0x^2 + 7x + 3}$

$-(3x^5 + 6x^4)$

$-1x^4 - 4x^3$

$-(-1x^4 - 2x^3)$

$-2x^3 + 0x^2$

$-(-2x^3 - 4x^2)$

$4x^2 + 7x$

$-(4x^2 + 8x)$

$-1x + 3$

$-(-1x - 2)$

5

$\frac{-1x^4}{x} = -x^3$

$\frac{4x^2}{x} = 4x$

(b) Find P(-2)

$P(-2) = 3(-2)^5 + 5(-2)^4 - 4(-2)^3 + 7(-2) + 3$

$P(-2) = 5$

Remainder Theorem:
If the polynomial $P(x)$ is divided by $x-c$, then the remainder is the value $P(c)$.

* set the divisor = 0 and solve for x

Plug that value into the function to get the remainder.

1. Let $P(x) = x^3 - 2x^2 + 3x - 1$. Find $P(3)$ using 2 different methods.

This is the value when we set the divisor = 0 (comes from x-3)

①

$$\begin{array}{r}
x^2 + x + 6 \\
x-3 \,\big)\, x^3 - 2x^2 + 3x - 1 \\
-(x^3 - 3x^2) \qquad\qquad \\
\hline
1x^2 + 3x \qquad \\
-(1x^2 - 3x) \qquad \\
\hline
6x - 1 \\
-(6x - 18) \\
\hline
17
\end{array}$$

② $P(3) = 3^3 - 2(3)^2 + 3(3) - 1$

$P(3) = 17$

Remainder when we divide is the value we get when we plug in the zero.

Factor Theorem:

A polynomial $P(x)$ has a factor of $x-c$ if and only if $P(c)=0$.

* If there is no remainder then the divisor is a factor. Plug the "zero" into the polynomial, if we get 0 then it is a factor.

2. Show that $x-2$ is a factor of $P(x)=x^3-3x^2+7x-10$.

$x-2=0$

$x=2$

Called a zero

$P(2)=2^3-3(2)^2+7(2)-10$

$P(2)=8-12+14-10$

$P(2)=0$

Because $P(2)=0$ then $(x-2)$ is a factor of $P(x)$

3. (a) Use the factor theorem to show that $x+3$ is a factor of $P(x)=x^3-x^2-8x+12$

$x+3=0$
$x=-3$ $\longrightarrow$ $P(-3)=(-3)^3-(-3)^2-8(-3)+12$

$P(-3)=0$

$(x+3)$ is a factor!

(b) Factor $P(x)$ completely.

4. Let $P(x) = x^3 - 7x + 6$.

(a) Show that $P(1) = 0$

(b) Factor $P(x)$ completely.

5. Find a polynomial of degree 4 that has zeros $-3, 0, 1,$ and 5.